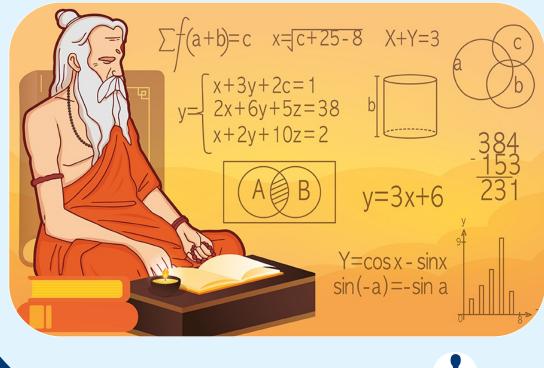


MATS CENTRE FOR OPEN & DISTANCE EDUCATION

Vedic Mathematics

Bachelor of Computer Applications (BCA) Semester - 3











Bachelor of Computer Applications ODL BCA VAC 003 Vedic Mathematics

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COURSE INTRODUCTION

Course Introduction:

This Vedic Mathematics course is designed to empower students with efficient and intuitive arithmetic techniques derived from ancient Indian mathematical wisdom. Using the powerful and elegant Sutras of Vedic Maths, learners will be able to perform mental calculations faster and more accurately. The course focuses on core concepts like addition, subtraction, digit sums, and innovative checking methods, alongside special multiplication techniques. Through practical application and a structured approach, students will develop a strong number sense and confidence in solving problems with speed and clarity.

Module 1: Fundamentals of Vedic Arithmetic

This module introduces the core concepts and foundational techniques of Vedic Mathematics. Students will explore easy and fast methods for **addition** and **subtraction**, enabling them to perform calculations with enhanced speed and accuracy. Key Sutras such as *"Nikhilam Navatashcaramam Dashatah"* and *"Ekadhikena Purvena"* will be introduced to build conceptual clarity and efficiency.

Module 2: Mastering Digit Sums and Checking Techniques This module dives into Digit Sums and various traditional methods for verifying the correctness of arithmetic operations. Techniques like Casting out 9s, the 9-Check Method, and the 11-Check Method provide logical tools to validate solutions quickly without redoing entire calculations.

Module 3: Special Multiplication Techniques

In this final module, students will learn **special multiplication methods** using unique Vedic Sutras that drastically simplify complex multiplication. These techniques enable learners to handle large numbers mentally, with ease and elegance, making them particularly useful for competitive exams and real-life problem-solving.

Module 4: Vedic Methods for Squaring and Division Explores mental squaring techniques and efficient division methods using Vedic patterns, enabling fast, elegant handling of roots and quotients.

MODULE 1 Fundamentals of Vedic Arithmetic

LEARNING OUTCOMES

By the end of this module, learners will be able to:

- Understand the fundamental principles of Vedic Mathematics and its historical roots.
- Perform quick and accurate addition and subtraction using Vedic techniques.
- Apply Sutras like Nikhilam Navatashcaramam Dashatah and Ekadhikena Purvena in basic arithmetic.
- Recognize how Vedic methods simplify mental calculations.
- Develop a strong foundation for advanced Vedic techniques through hands-on practice.



Unit 1: Addition

1.1 Addition - Completing the whole

Addition is the most basic operation and adding number 1 to the previous number generates all the numbers. The Sutra "By one more than the previous one describes the generation of numbers from unity.

Completing the whole method

The VEDIC Sutra 'By the Deficiency' relates our natural ability to see how much something differs from wholeness.

7 close to 10

- 8 close to 10
- 9 close to 10
- 17,18,19, are close to 20

27, 28, 29, are close to 30

37, 38, 39, are close to 40

47, 48, 49, are close to 50

57, 58, 59, are close to 60

67, 68, 69, are close to 70

77, 78, 79, are close to 80

87, 88, 89, are close to 90

97, 98, 99, are close to 100 and so on.

The ten Point Circle

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Notes

We can easily say that 9 is close to 10, 19 is close to 20 etc. We can use this closeness to find addition and subtraction.

The ten Point Circle **Rule:** By completion noncompletion 9 Five number pairs 1+9 2 + 88 3 + 72 4 + 65 + 5Use these number pairs to make groups of '10' when adding numbers. 5 **Example:** 24 + 26 = 20 + 4 + 20 + 6= 20 + 20 + 10 = 50Below a multiple of ten **Rule: By the deficiency** 49 is close to 50 and is 1 short. 38 is close to 40 and is 2 short. **Example:** 59 + 4 = 59 + 1 + 3 = 60 + 3 = 63 {59 is close to 60 and 1 short 50, 59 + 4 is 60} **Example:** 38 + 24 = 38 + 2 + 22 = 40 + 22 = 62or 38 + 24 = 40 + 24 - 2 = 64 - 2 = 62 {38 is close and is 2 sheet so, 38 + 24 is 2 short from 40 + 24 hence 38 + 24 = 40 + 24-2 = 64 - 2 = 62Example Add 39 + 6 = ?39 is close to 40 and is 1 less then it. So we take 1 from the 6 to make up 40 and then we have 5 more to add on which gives 45 Add

 $29 + 18 + 3 \qquad [As 3 = 1 + 2 and 29 + 1 = 30, 18 + 2 = 20]$ 29 + 18 + 1 + 2



30 + 20 = 50 Note we break 3 into 1 + 2 because 29 need 1 to become 30 and 18 need 2 become 20] Add

39 + 8 + 1 + 439 + 8 + 1 + 2 + 240 + 10 + 2 = 52

Sum of Ten

The tenpoint circle illustrates the pairs of numbers whose sum is 10. Remember: There are eight unique groups of three number that sum to 10, for example 1 + 2 + 7 = 10



Can you find the other seven groups of three number summing to 10 as one example given for you?



Adding a list of numbers

Rule: By completion or non-completion Look for number pairs that make a multiple of $10\ 7+6+3+4$ The list can be sequentially added as follows: 7+6=13 then 13+3=16 then 16+4=20Or

You could look for number pairs that make multiples of 10. 7 + 3 is 10 and 6 + 4 is 10 hence 10 + 10 is 20.

Similarly: 48 + 16 + 61 + 32 = (48 + 32) + (16 + 1 + 60)= 80 + 77 = 157

or

7 + 8 + 9 + 2 + 3 + 5 + 3 + 1 + 2 + 3 + 7 + 9

10 10 = 10 + 10 + 10 + 10 + 10 + 9 = 59



PRACTICE PROBLEMS

Add by using completing the whole method

1.	39 + 8 + 1 + 5 =	2. 18 + 3 + 2 + 17 =
3.	9+41+11+2=	4. 47 + 7 + 33 23 =
5.	23 + 26 + 27 + 34 =	6. 22 + 36 + 44 + 18=
7.	33 + 35 + 27 + 25=	8. 18 + 13 + 14 + 23=
9.	3+9+8+5+7+1+2=	10. $37 + 25 + 33 =$
11.	43 + 8 + 19 + 11=	12. $42 + 15 + 8 + 4 =$
13.	24 + 7 + 8 + 6 + 13 =	14. $16 + 43 + 14 + 7 =$
15.	13 + 38 + 27 =	

ADDITION

Completing the whole method (class VI commutative & associative property)

1.	39 + 17 + 11 + 13 =	2.	16 + 23 + 24 + 7 =
3.	12 + 51 + 9 + 18 =	4.	35 + 12 + 55 =
5.	123 + 118 + 27 =	6.	35 + 15 + 16 + 25=
7.	58 + 41 + 12 + 9=	8.	223 + 112 + 27 =
9.	24 + 106 + 508 + 12=	10.	506 + 222 + 278 =

1.2 Adding from left to right

The conventional methods of mathematics teachers use to do calculation from right and working towards the left.

In Vedic mathematics we can do addition from left to right which is more, useful, easier and sometimes quicker.

Add from left to right

1.	23	2	•	234
	+ 15			+524
	38			758
3.	15	4	•	235
	<u>38</u>			<u>526</u>
	43			751
	Add 1			Add 1
	= 53			= 761



The method: This is easy enough to do mentally, we add the first column and increase this by 1 if there is carry coming over from the second column. Then we tag the last figure of the second column onto this

Mental math

Add from left to right

(1) 6 6 <u>+ 5 5</u>	(2) 5 4 6 + 67 1	(3) 5 3 4 + 7 1 7	(4) 1 4 5 7 + 2 8 5 7
(5) $45 + 76$	(6) $3 1 2 4 6 5$ + 7 6 1 2 4 6	(7) 7 4 5 + 2 7	$\begin{array}{cccc} (8) & 1 & 4 & 3 & 2 \\ + & 8 & 6 & 6 & 8 \end{array}$
(9) 8 5	(10) 537	(11) 4 5 6	(12) 2648
(3) (3)	(10) $\frac{+718}{-118}$	(11) 450 +127	(12) 204 8 +8365
(13) 1345	(14) 546	(15) 7885	(16) 378
+5836	+4561	+1543	+ 48
(17) 35671	(18) 2468		
+12345	+ 123		

1.3 Shudh method for a list of number

Shudh means pure. The pure numbers are the single digit numbers i.e. 0, 1, 2, 3...9. In Shudh method of addition we drop the 1 at the tens place and carry only the single digit forward.

Example: Find 2 + 7 + 8 + 9 + 6 + 4

We start adding from bottom to top because that is how our eyes naturally move but it is not necessary we can start from top to bottom.



As soon as we come across a two-digit number, we put a dot instead of one and carry only the single digit forward for further addition. We put down the single digit (6 in this case) that we get in the end. For the first digit, we add all the dots (3 in this case) and write it.

Adding two- or three-digit numbers list

We start from the bottom of the right most columns and get a single digit 6 at the unit place. There are two dots so we add two to the first number (4) of the second column and proceed as before. The one dot of this column is added to the next and in the end, we just put 1 down (for one dot) as the first digit of the answer.

. 23.4 6.5.8 .81.8 <u>46</u> 1756 (Shudh method)

•5	26
•9	• 4•5
4	34
•6	•8 1
7	52
•8	23 8
4	
43	

Add the following by (Shudh method)

1.	5	2.	37	3.	345
	7		64		367
	6		89		289
	8		26		+ 167
	4		+ 71		
+	9				



	4.	3126	5.	468	6.	235
		1245		937		579
		4682		386		864
		+ 5193	_	654	_	+ 179
	7.	59	8. 4	49	9.	98
		63	(53		83
		75	7	78		78
		82	8	35		62
		+91	+ 9	97		+ 44
10.	37		11. 24	461	12.	9721
	79		46	585		2135
	52		62	203		5678
	88		12	234		207
	+ 91		+ 5	432		+ 1237

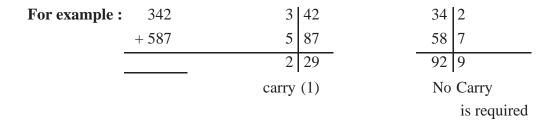
Number Spliting Method

Quick mental calculations can be performed more easily if the numbers are 'split into more manageable parts.

For example: Split into two more manageable sums

+ 3642	36	42	<i>Note:</i> The split allows us to
			add 36 + 24
2439	+ 24	39	and $42 + 39$ both of which can
			be done
	60	81	mentally

Remember : Think about where to place the split line. It's often best to avoid number 'carries' over the line.



A carry of '1' over the line is required

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Unit 2: Subtraction

Sutra: All from 9 and the Last from 10

The Concept of Base Numbers made up of only 1's and 0's are known as a Base. Examples of a Base are 10, 100, 1000, 1, .01....etc The base method is used for subtracting, multiplying or dividing numbers. Like 98, 898, 78999 etc that are close to base. Applying the formula "All form 9 and Last form 10" to any number especially the big one's reduces it to its smaller Counterpart that can be easily used for calculations involving the big digits like 7, 8, and 9. Applying the formula "All from 9 and the last from 10" Example: Apply 'All from 9 Last from 10' to Subtract 789 from 1000

789 $\downarrow \downarrow \downarrow$ 211

[Here all from 9 last from 10 means subtract 78 8 from 9 and from 10, so weget 211]

We get 211, because we take 7 and 8 from 9 and 9 from 10.

from 10000	from 100	from 100	from 100000
2772	54	97	10804
$\downarrow \downarrow \downarrow \downarrow \downarrow$	$\downarrow\downarrow$	$\downarrow\downarrow$	$\downarrow \downarrow \downarrow \downarrow \downarrow \downarrow \downarrow$
7228	46	03	89196

If you look carefully at the pairs of numbers in the above numbers you may notice that in every case the total of two numbers is a base number 10, 100, 1000 etc. This gives us an easy way to subtract from base numbers like 10, 100, 1000......

Subtracting from a Base

Example: - 1000 - 784 = 216

Just apply 'All from 9 and the Last from 10' to 784, difference of 7 from 9 is 2, 8 from 9 is 1, 4 from 10 is 6 so we get 216 after subtraction.



When subtracting a number from a power of 10 subtract all digits from

1000	_	from 9	from 10
- 276			•
724		276	5
		$\downarrow\downarrow\downarrow\downarrow$	
		724	

9 and last from 10.

Subtracting from a Multiple of a Base

Sutra: 'All from 9 and the last from 10'and

'One less than the one before'

Example: 600 – 87

We have 600 instead of 100. The 6 is reduced by one to 5, and the All from 9 and last from 10 is applied to 87 to give 13. Infact, 87 will come from one of those six hundred, so that 500 will be left.

600 - 87 = 513 [Note: First subtract form 100 then add 500, as 500 + 13 = 513]

Example: Find 5000 – 234

5, is reduced to one to get 4 and the formula converts 234 to 766

5000-234=4766

Example: 1000 - 408 = 592**Example:** 100 - 89 = 11



Example: 1000 - 470 = 530 [Remember apply the formula just to 47 here.]

If the number ends in zero, use the last non-zero number non-zero number as the last number for example.

Hence 1000 - 4250 = 5750

Adding Zeroes

In all the above sums you may have noticed that the number of zeros in the first number is the same as the numbers of digits in the number being subtracted.



Example: 1000 - 53 here 1000 has 3 zeros and 53 has two digits. We can solve this by writing

We put on the extra zero in front of 53 and then apply the formula to 053.

Example: 10000 – 68, Here we need to add two zeros. 10000 – 0068 = 9932

Practice Problems Subtract from left to right

- (1) 86 27 = (2) 71 34 =
- $(3) \quad 93 36 = (4) \quad 55 37 =$
- $(5) \qquad 874 567 = (6) \qquad 804 438 =$
- $(7) \qquad 793 627 = (8) \qquad 5495 3887 =$
- 9) 9275 1627 = (10) 874 579 =
- $(11) \quad 926 624 = (12) \quad 854 57 =$
- $(13) \quad 8476 6278 = (14) \quad 9436 3438 =$

Subtract the following mentally

- (1) 55-29 = (2) 82-558 =
- (3) 1000 909 = (4) 10000 9987 =
- (5) 10000 72 = (6) 50000 5445 =
- $(7) \qquad 70000 9023 = \qquad (8) \ 30000 387 =$
- $(9) \qquad 46678 22939 = (10) 555 294 =$
- $(11) \quad 8118 1771 = (12) \ 61016 27896 =$

Example: Find 9000 – 5432

Sutra: 'One more than the previous one' and 'all from 9 and the Last from the 10'



Considering the thousands 9 will be reduced by 6 (one more than 5) because we are taking more than 5 thousand away

'All from 9 and the last from 10' is than applied to 432 to give 568 9000 -5432 = 3568

Similary—7000 – 3884

= 3116 $\{3 = 7 - 4, 4 \text{ is one more than } 3 \text{ and } 116 = 4000 - 3884\}$ by all from a and the last from 10 $\}$

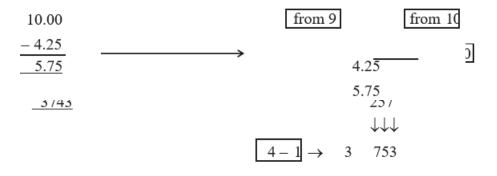


If the number is less digits, then append zero the start : 1000

When subtracting form a multiple of a power of 10, just decrement the first digit by 1, then subtract remaining digits :

Look at one more example:

Money: A great application of "all from 9 and last from 10" is money. Change can be calculated by applying this sutra mentally for example:



This is helpful because most our rupee notes are multiple of 10's.

PRACTICE PROBLEMS

Subtract (base method)



Notes

(1) $1000 - 666$	(2) 10000 - 3632
------------------	------------------

- (3) 100 54
- (4) 100000 16134 (6) 1000 - 840(5) 1000000 - 123456
- (7) 1000 88(8) 10000 - 568
 - (10) 100000 5542
- (9) 1000 61(11) 10000 - 561(12) 10000 - 670

Subtract (multiple of base)

- (1) 600 72 =(2) 90000 - 8479 =
- (3) 9000 758 =(4) 4000 - 2543 =
- (5) 7000 89 =(6) 300000 - 239 =
- (7) 1 0.6081 =(8) 5 - 0.99 =

Subtracting Near a base

Rule: By completion or non completion.

when subtracting a number close to a multiple of 10. Just subtract from the multiple of 10 and correct the answer accordingly.

Example: 53 – 29

29 is just close to 30, just 1 short, so subtaract 30 from 53 making 23, then add 1 to make 24

53 - 29 = 53 - 30 + 1= 23 + 124 = 45 - 18= 45 - 20 + 225 + 2= 27 {18 is near to 20, just 2 short) =

Use the base method of calculating

To find balance

Q. Suppose you buy a vegetable for Rs. 8.53 and you buy with a Rs. 10 note. How much change would you expect to get? Ans. You just apply "All from 9 and the last from 10" to 853 to get 1.47.



Q. What change would expect from Rs. 20 when paying Rs. 2.56? Ans.The change you expect to get is Rs. 17.44 because Rs. 2.56 from Rs.10 is Rs. 7.44 and there is Rs. 10 to add to this.

Practice Problem

Q1. Rs. 10 – Rs. 3.45 Q2. Rs. 10 – Rs. 7.61 Q3. Rs. 1000 – Rs. 436.82 Q4. Rs. 100 – Rs. 39.08

Subtracting number just below the base

Example: find 55 - 29Subtraction of numbers using "complete the whole" Step 1: 20 is the sub base close to 19 19 is 1 below 20 Step 2: take 20 from 55 (to get 35) Step 3: Add 1 back on 55 - 19 = 36Example 61 - 3838 is near to 40 = 40 - 38 = 261 - 40 = 2161 - 38 = 21 + 2 = 23

Example

44 - 19 19 + 1 = 20 44 - 20 = 2444 - 19 = 24 - 1 = 23

Example

88 - 49 49 + 1 = 50 88 - 50 = 3888 - 49 = 38 + 1 = 39

Example

55 - 1717 + 3 = 2055 - 20 = 35



55 - 17 = 35 + 3 = 38

Number spliting Method

As you have use this method in addition the same can be done for subtraction also:

+ 3642		36	42
2439	\longrightarrow	+ 24	39
		12	03

Note: The split allows on to add '36 - 24'and 42 - 39 both of which can be do mentally

General Method of subtraction

Subtraction from left to right

In this section we show a very easy method of subtracting numbers from left to right that we have probably not seen before. We start from the left, subtract, and write it down if the subtraction in the next column can be done. If it cannot be done you put down one less and carry 1, and then subtract in the second column.

Subtraction from left to right. Example

Find (3)	(4)	Find	(5)	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	_	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		

Left to right



Starting from the left we subtract in each column 3-1=2 but before we put 2 down we check that in next column the top number is larger. In this case 5 is larger than 1 so we put 2 down

In the next column we have 5-1=4, but looking in the third column we see the top number is not larger than the bottom(5 is less than 8) so instead putting 4 down we put 3 and the other 1 is placed as the flag, as shown so that 5 becomes 15, so now we have 15-8=7. Checking in the next column we can put this down because 6 is greater than 2. In the fourth column we have 6-2=4, but looking at the next column (7 is smaller than 8) we put down only 3 and put the other flag with 7 as shown finally in the last column 17-8=9.



MODULE 2 Mastering Digit Sums and Checking Techniques

LEARNING OUTCOMES

By the end of this module, learners will be able to:

- Grasp the concept and application of digit sums in verifying arithmetic operations.
- Use techniques such as Casting Out 9s, the 9-Check, and the 11-Check method for error detection.
- Enhance accuracy in problem-solving through logical verification tools.
- Validate answers without redoing the entire calculation.
- Cultivate confidence in self-assessing mathematical work.



Unit 3: Digit Sums and Casting out 9s

The word digit means a single figure number: The numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 are all digits. Big numbers can be reduced to single digit by adding the constituents.

Digit Sums

A digit sum is the sum of all the digits of a number and is found by adding all of the digits of a number The digit sum of 35 is 3 + 5 = 8The digit sum of 142 is 1 + 4 + 2 = 7

Note: If the sum of the digits is greater than 9, then sum the digits of the result again until the result is less than 10.

greater than 9, so need to add again

The digit of 57 is 57 is 57 is 5 + 7 = $12 \rightarrow 1 + 2 = 3$ Hence the digit sum of 57 is 3.

The digit sum of 687 is 687 is $6+8+7=21 \rightarrow 2+4=3$ Hence the digit sum of 687 is 3.

Keep findig the digit sum of the result + unitl it's less then 10
0 and 9 are requivalent

Look and undevstand some more example:

To find the digit sum of 18, for the example we just add 1 and 8, i.e. 1 + 8 = 9 so the digit sum of 18 is 9. And the digit sum of 234 is 9 because 2 + 3 + 4 = 9

Following table shows how to get the digit sum of the following members

Sometimes two steps are needed to find a digit sum.

So for the digit sum of 29 we add 2 + 9 = 11 but since 11 is a 2-digit number we add again 1+1=1 So for the digit sum of 29 we can write



Notes

29 = 2 + 9 = 11 = 1 + 1 = 1

Similarity for 49 = 4 + 9 = 13 = 1 + 3 = 4 So the digit sum of 49 is 4.

Number 14 Digit sum 1 + 4 = 5 Single digit 5 19 1 + 9 = 10 1

Number 14	Digit sum 1 + 4 = 5	Single digit 5
19	1 + 9 = 10	1
39	3+9 = 12	3
58	5 + 8 = 13	4
407	4 + 0 + 7 = 11	2

CASTING OUT NINE

Adding 9 to a number does not affect its digit sum So 5, 59, 95, 959 all have digit sum of 5.

For example to find out the digit sum of 4939 we can cast out nines and just add up the 3 and 4 so digit sum is 7 or using the longer method we add all digit 4 + 9 + 3 + 9 = 25 = 2 + 5 = 7

There is another way of casting out the nines from number when you are finding its digit sum. Casting out of 9's and digit totalling 9 comes under the Sutra when the samuccaya is the same it is zero.

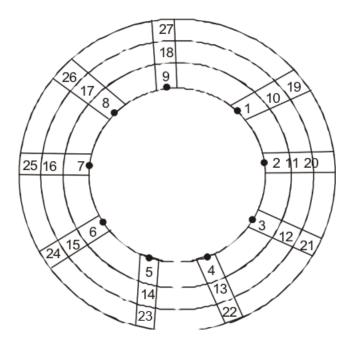
So in 465 as 4 and 5 total nine, they are cast out and the digit sum is 6: when the total is the same

(as 9) it is zero (can be cast out) cancelling a common factor in a fraction is another example.



Number	Digit sum
1326	3
25271	8
9643	4
23674	4
128541	3
1275	6
6317892	9 or 0

Nine Point Circle



Number at each point on the circle have the same digit sum. By casting out 9's, finding a digit sum can be done more quickly and mentally.

9. Check Method

Digit sum can be used to check that the answers are correct.

Example: Find 23 + 21 and check the answer using the digit sums

23 = digit sum of 23 is 2 + 3 = 5+21 = digit sum of 21 is 2 + 1 = 3

44 = digit sum of 44 is 4 + 4 = 8



Notes

If the sum has been done correctly, the digit sum of the answer should also be 8 Digit sum of 44=8 so according to this check the answer is probably correct.

There are four steps to use digit sum to check the answers:

- 1. Do the sum.
- 2. Write down the digit sums of the numbers being added.
- 3. Add the digit sums.
- Check whether the two answers are same in digit sums.
 Add 278 and 119 and check the answer
 278

270

+119

397

1. We get 397 for the answer

2. We find the digit sum of 278 and 119 which are, 8 and 2 respectively

- 3. Adding 8 and 2 gives 10, digits sum of 10=1+0=1
- 4. Digit sum of 397 is
- 3 + 9 + 7 = 19 = 1 + 9 = 10 = 1 + 0 = 1

Which confirm the answer?

CAUTION!

Check the following sum:

Here an estimation can help you to find the result more accurate if by mistage you write 400 in place of 490 then it will show the result is correct.

The check is 9 + 4 = 13 = 4 which is same as the digit sum of the answer which confirms the answer. However, if we check the addition of the original number we will find that it is incorrect! This shows that the digit sum does not always find errors. It usually works but not always. We will be looking at another checking device i.e. 11 - check method. **Note:** The difference of 9 and its multiples in the answer make errors. So, keep in mind a rough estimation.

Practice Problems



Digit sum Puzzles

1. The digit sums of a two digit number is 8 and figures are the same, what is the number?

2. The digit sum of a two-digit number is 9 and the first figure is twice the second, what is it?

3. Give three two-digit numbers that have a digit sum of 3.

4.A two-digit number has a digit sum of 5 and the figures are the same. What is the number?

5.Use casting out 9's to find the digit sums of the numbers below.

Number	
465	
274	
3456	
7819	
86753	
4017	
59	

6. Add the following and check your answer using digit sum check

- (1) 66 + 77 = (2) 57 + 34 =
- $(3) \quad 94 + 89 = \qquad (4) \quad 304 + 233 =$
- $(5) \quad 787 + 132 = \qquad (6) \quad 389 + 414 =$
- (7) 5131 + 5432 = (8) 456 + 654 =



Unit 4: 11-Check method

We have already used the digit sum check that helps to show if a calculation is correct. This method works because adding the digit in a number gives the remainder of the number after division by 9. A similar method works by using remainders of numbers after division by 11 rather than 9

Alternate digit sum or Eleven-check Method

Suppose we want another check for $2434 \times 23 = 55982$ it can be done in the following steps

Step1: Alternately add and subtract (starting from right moving towards left) the digits of each numbers as described below

Step 2: Now multiply the Digit Sum to get the product $3 \times 1 = 3$ Since

Number	Alternating signs	Digit sum
2434	-2 + 4 - 3 + 4	3
23	-2 + 3	1
55982	+5 - 5 + 9 - 8 + 2	3

the Digit Sum of the product and the two numbers is the same, the answer is correct as per 11 check method.

Two digit and Negative number in the digit sum checking the sum of addition.

4364 + 1616 Left to right 4364 <u>1916</u> 6280



Number	Alternating signs	Digit sum	Single digit
4364	-4 + 3 - 6 + 4	-3 (11-3)	8
1916	-1 + 9 - 1 + 6	13(11+2)	2
6280	-6 + 2 - 8 + 0	- 12	10
		11 - 12 = -1	
		11 - 1 = 10	

Step3: Apply the following rules to get a single positive digit for the number

• Subtract the negative numbers below 11 from 11 to get its positive counterpart so -3 = 11 - 3 = 8 And -12 = -12 + 11 = -1 = 11 - 1 = 10

• For the two digit number above 11, divide the number by 11 and get the remainder as the positive digit sum so $13 \div 13$ gives remainder 2. Alternately, adding and subtracting digit of 13 starting from right can obtain this same result.

Step 4: now add the Digit sums to get the sum 8 + 2 = 10, the answer is correct as per 11 check method.

Two digits in the digit sum Check subtraction problem 2819174 - 839472 2819174 839472 1979702

Step 1: Alternatively add and subtract (staring from right moving towards left) the digit of each numbers as described below

Step 2: Apply	Number	Alternating signs	Digit sum	Single digit
the following rules to get a	2819174	+2-8+1-9+1- 7+4	-16	11–5=6
single positive			(-16+11=-5)	
digit for the	839472	-8+3-9+4-7+2	-15 (-15+11=-4)	11-4=7
number • The	1979702	+1-9+7-9+7- 0+2	-1	11-1=10

negative

numbers below -11 are to be first divided by 11 to get the remainder.



Notes

Than subtract the remainder from 11 to get its positive counterpart. So -16/11 Remainder is -5 and -5 = 11 - 5 = 6 similarly -15/11 Remainder -4 = 11 - 4 = 7.

The negative number -1 = 11 - 1 = 10

Step3: Now subtract the Digit sums to get the answer 6 - 7 = -1 = 10, the answer is correct as per 11- checked method.

Practice Problems

•

Get the digit sum and single digit for the following numbers.

Numbers	Alternative signs	Digit sums	Single digit
567			
1536			
93823			
1978712			
849391			
82918			
5949393			
176780			

Using 11 check method check the following Addition problems:

- (1) 37 + 47 = 84
- (2) 55 + 28 = 83
- (3) 47 + 25 = 72
- (4) 29 + 36 = 65
- $(5)\ 526 + 125 = 651$
- (6) 1328 + 2326 = 3654
- (7) 129 + 35644 = 35773
- $(8) \ 3425 + 7491 + 8834 = 19750$
- (9) 1423178 + 5467 + 123 + 34 = 1428802
- (10) 1314 + 5345 + 65 + 781 = 7505

Check the following subtraction problems: (1) 63 - 28 = 35

- (2) 813 345 = 468
- (3) 695 368 = 372
- $(4) \ 3456 281 = 3175$
- (5) 7117 1771 = 5346
- (6) 8008 3839 = 4165



(7) 6363 - 3388 = 2795(8) 51015 - 27986 = 23029(9) 14285 - 7148 = 7137(10) 9630369 - 3690963 = 5939406



MODULE 3 Special Multiplication Techniques

LEARNING OUTCOMES

By the end of this module, learners will be able to:

- Apply base and sub-base multiplication methods for numbers near powers of 10.
- Use vinculum and complementary number strategies to simplify calculations.
- Multiply two-, three-, and four-digit numbers mentally from right to left.
- Identify and apply suitable Vedic Sutras for different multiplication scenarios.
- Improve speed and accuracy in complex multiplication tasks.



Unit 5: Base and Sub-Base Methods

Multiplication in considered as one of the most difficult of the four mathematical operations. Students are scared of multiplication as well as tables. Just by knowing tables up to 5 students can multiply bigger numbers easily by some special multiplication methods of Vedic Mathematics. We should learn and encourage children to look at the special properties of each problem in order to understand it and decide the best way to solve the problem. In this way we also enhance the analytical ability of a child. Various methods of solving the questions /problems keep away the monotonous and charge up student's mind to try new ways and in turn sharpen their brains.

Easy way for multiplication

Sutra: Vertically and Cross wise:

For speed and accuracy tables are considered to be very important. Also students think why to do lengthy calculations manually when we can do them faster by calculators. So friends/ teachers we have to take up this challenge and give our students something which is more interesting and also faster than a calculator. Of course it's us (the teachers/parents) who do understand that more we use our brain, more alert and active we will be for, that is the only exercise we have for our brain.

Example 1: 7 x8

Step 1: Here base is 10,

7-3 (7 is 3 below 10) also called deficiencies

 \times 8 – 2 (8 is 2 below 10) also called deficiencies

Step 2: Cross subtract to get first figure (or digit) of the answer:

7-2=5 or 8-3=5, the two difference are always same.

Step 3 : Multiply vertically *i.e.* $-3 \times -2 = 6$ which is second part of the answer. So,

$$7-3$$

8-2 *i.e.* $7 \times 8 = 56$
5/6

Example 2: To find 6×7

Step 1 : Here base is 10,

6-4 (6 is 4 less than 10) *i.e.* deficiencies

```
7-3 (7 is 3 less than 10) i.e. deficiencies
```

Step 2: Cross subtraction : 6 - 3 = 3 or 7 - 4 = 3 (both same)

Step 3: $-3 \times -4 = +12$, but 12 is 2 digit number so we carry this 1

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Notes

over to 3 (obtained in 2 step)

$$6-4$$

$$\frac{7-3}{3/(1)} 2 i.e. \ 6 \times 7 = 42$$

Try these: (1) 9×7 (ii) 8×9 (iii) 6×9 (iv) 8×6 (v) 7×7

Second Method:

Same Base Method :

When both the numbers are more than the same base. This method is extension of the above method i.e. we are going to use same sutra here and applying it to larger numbers.

Example 1: 12 × 14

Step 1: Here base is 10

12 + 2 [12 is 2 more than 10 also called surplus]

14 + 4 [14 is 4 more than 10also called surplus]

- Step 2: Cross add: 12 + 4 = 16 or 14 + 2 = 16,(both same) which gives first part of answer = 16
- **Step 3**: Vertical multiplication: $2 \times 4 = 8$

So,

12 + 2 14 + 4 $16 / 8So, 12 \times 14 = 168$ (14 + 2 = 12 + 4)

Example 2:105x 107

Step1: Here base is 100

- 105 + 05 [105 is 5 more than 100 or 5 is surplus]
- 107 + 07 [107 is 7 more than 100 or 7 is surplus]

Base here is 100 so we will write 05 in place of 5 and 07 in place of 7

- **Step 2**: Cross add: 105 + 7 = 112 or 107 + 5 = 112 which gives first part of the answer = 112
- Step 3: Vertical multiplication: $05 \times 07 = 35$ (two digits are allowed) As the base in this problem is 100 so two digits are allowed in the second part. So, $105 \times 107 = 11235$

Example 3: 112 x 115

Step 1: Here base is 100

112 + 12 [2 more than 100 i.e. 12 is surplus]

- 115 + 15 [15 more than 100 i.e. 15 is surplus]
- **Step 2**: Cross add: 112 + 15 = 127 = 115 + 12 to get first part of



answer

i.e.127

- Step 3: Vertical multiplication $12 \times 15 = ?$ Oh, my god!It's such a big number. How to get product of this? Again use the same method to get the product.
 - 12 + 2
 - 15 + 5

 $12 + 5 = 15 + 2 = 17/(1) 0, 17 + 1/0 = 180 i.e. 12 \times 15 = 180$ But only two digits are allowed here, so 1 is added to 127 and we get (127 + 1) = 128 So, $112 \times 115 = 128, 80$

Try these: (i)12 × 14 (ii) 14 × 17 (iii) 17 × 19 (iv) 19 × 11 (v) 11 × 16 (vi) 112 × 113 (vii) 113 × 117(viii) 117 × 111 (ix) 105 × 109 (x) 109 × 102 (xi) 105 × 108 (xii) 108 × 102 (xiii) 102 × 112 (xiv) 112 × 119 (xv) 102 × 115

Both numbers less than the same base:

Same sutra applied to bigger numbers which are less than the same base.

Example1: 99 × 98

- **Step 1**: Check the base: Here base is 100 so we are allowed to have two digits on the right hand side.
 - \therefore 99 01 (1 less than 100) i.e. 01 deficiency

98-02 (2 less than 100) i.e. 0 2 deficiency

- Step 2: Cross subtract: 99 02 = 97 = 98 01 both same so first part of answer is 97
- **Step3**: Multiply vertically $-01 \times -02 = 02$ (As base is 100 so two digits are allowed in second part So, $99 \times 98 = 9702$

Example 2 : 89 × 88

Step1: Here base is 100

So, 89 - 11 (i.e. deficiency = 11)

88 - 12 (i.e. deficiency = 12)

Step2: Cross subtract: 89 – 12 = 77 = 88 – 11(**both same**)

So, first part of answer can be 77

Step 3:Multiply vertically -11×-12

Again to multiply 11×12 apply same rule

11 + 1 (10 + 1)

12+2 (10+2)



Notes

 $11 + 2 = 13 = 12 + 1 / 1 \times 2 = 12$ so, $11 \times 12 = (1)$ 32 as only two digits are allowed on right hand side so add 1to L.H.S. So, L.H.S. = 77 + 1 = 78Hence $89 \times 88 = 7832$ **Example 3**: 988 × 999 Step 1: As the numbers are near 1000 so the base here is 1000 and

hence three digits allowed on the right hand side 988 - 012(012 less than 1000) i.e. deficiency = 0.12

999 - 001 (001 less than 1000) i.e. deficiency = 00 1

Step 2: Cross – subtraction: 988 – 001 = 987 = 999 – 012 = 987 So first part of answer can be 987

Step 3: Multiply vertically: -012 xs - 001 = 012 (three digits allowed) $988 \times 999 = 987012$...

How to check whether the solution is correct or not by 9 - check method.

Example 1: $99 \times 98 = 9702$ Using 9 – check method.

As,
$$99 = 0$$
 Product (L.H.S.) = $0 \times 8 = 0$ [taking $9 = 0$]
 $98 = 8$

R.H.S. = $97|02 = 7 + 2 = 9 \neq 0$ 9702 = 9 both are same

As both the sides are equal answer may be correct.

Example 2: 89 × 88 = 7832

 $89 \rightarrow 8 + 9 = 17 \rightarrow 1 + 7 = 8$ $88 \rightarrow 8 + 8 = 16 \rightarrow 1 + 6 = 7$

L.H.S. Check:

 $8 \times 7 = 56 \longrightarrow 5 + 6 = 11 \longrightarrow 1 + 1 = 2$

R.H.S. = 7832

 $7 + 8 + 3 + 2 = 20 \rightarrow 2 + 0 = 2$

LHS = RHS = 2, so the answer may be correct.

```
Example 3: 988 × 999 = 987012
```

 $988 \rightarrow 9 + 8 + 8 = 25 \rightarrow 2 + 5 = 7$ $999 \rightarrow 9 + 9 + 9 = 27 \rightarrow 2 + 7 = 9 \rightarrow 0$

L.H.S. Check:

 $7 \times 0 = 0$ **R.H.S.** = 987012 $9 + 8 + 7 + 0 + 1 + 2 = 27 \rightarrow 2 + 7 = 9 \rightarrow 0$ LHS = RHS = 0, so the answer may be correct.



Try These:

(i) 97 × 99 (ii) 89 × 89 (iii) 94 × 97 (iv) 89 × 92 (v) 93 × 95 (vi) 987 ×
998 (vii) 997 × 988 (viii) 988× 996 (ix) 983 × 998 (x) 877 × 996 (xi)
993 × 994 (xii) 789 × 993 (xiii) 9999 × 998 (xiv) 7897 × 9997
(xv) 8987 × 9996.

Multiplying bigger numbers close to a base: (number less than base) **Example 1**: 87798 x 99995

Step1: Base here is 100000 so five digits are allowed in R.H.S. 87798 - 12202 (12202 less than 100000) deficiency is 12202 99995 - 00005(00005 less than 100000) deficiency is 5 Step 2: Cross – subtraction: 87798 -00005 =87793 Also 99995 - 12202 = 87793 (both same) So first part of answer can be 87793 **Step 2 : Multiply vertically:** $-12202 \times -00005 = +61010$ $\therefore 87798 \times 99995 = 8779361010$ **Checking:** 87798 total 8 + 7 + 7 + 8 = 30 = 3 (single digit)99995 total = 5 $LHS = 3 \ge 5 = 15$ total = 1 + 5 = 6RHS = product = 8779361010 total = 15 = 1 + 5 = 6L.H.S = R.H.S. So, correct answer **Example 2 :** 88777 × 99997 Step 1: Base have is 100000 so five digits are allowed in R.H.S. 88777 - 11223i.e. deficiency is 11223 99997 - 00003 i.e. deficiency is 3 **Step 2**: Cross subtraction: 88777 – 00003 = 88774 = 99997 – 11223 So first part of answer is 88774 **Step 3**: Multiply vertically: $-11223 \times -00003 = +33669$ 88777 × 99997 = 8877433669 *.*.. **Checking:** 88777 total 8 + 8 + 7 + 7 + 7 = 37 = +10 = 199997 total = 7 $LHS = 1 \times 7 = 7$ RHS = 8877433669 = 8 + 8 + 7 + 7 + 4 = 34 = 3 + 4 = 7 i.e. LHS = RHS So, correct answer **Try These:**



(i) 999995 × 739984 (ii) 99837 × 99995 (iii) 99998 × 77338 (iv) 98456 Notes × 99993 (v) 99994 × 84321 Multiply bigger number close to base (numbers more than base) **Example 1**: 10021 × 10003 Step 1: Here base is 10000 so four digits are allowed 10021 + 0021 (Surplus) 10003 + 0003 (Surplus) Step 2: Cross – addition 10021 + 0003 = 10024 = 10003 + 0021 (both same) *.*.. First part of the answer may be 10024 **Step 3**: Multiply vertically: $10021 \times 0003 = 0063$ which form second part of the answer $10021 \times 10002 = 100240063$ **Checking:** 10021 = 1 + 2 + 1 + 1 = 410003 = 1 + 3 = 4LHS = $4 \times 4 = 16 = 1 + 6 = 7$ *.*.. RHS = 100240063 = 1 + 2 + 4 = 7As LHS = RHS So, answer is correct **Example 2**: 11123 × 10003 Step 1: Here base is 10000 so four digits are allowed in RHS 11123 + 1123 (surplus) 10003 + 0003 (surplus) Step 2: Cross – addition: 11123 + 0003 = 11126 = 10003 + 1123 (both equal) *.*.. First part of answer is 11126 Step 3: Multiply vertically: $1123 \times 0003 = 3369$ which form second part of answer $11123 \times 10003 = 111263369$ ·. Checking: 11123 = 1 + 1 + 1 + 2 + 3 = 810003 = 1 + 3 = 4 and $4 \times 8 = 32 = 3 + 2 = 5$ LHS = 5*.*.. R/H/S = 111263369 = 1 + 1 + 1 + 2 = 5As L.H.S = R.H.S So, answer is correct **Try These:** (i) 10004 × 11113 (ii) 12345 × 111523 (iii) 11237 × 10002 (iv) 100002 × 111523 (v) 10233 × 10005



Numbers near different base: (Both numbers below base)

Example 1: 98 × 9

Step 1: 98 Here base is 100 deficiency = 02

9 Base is 10 deficiency = 1

 \therefore 98 – 02 Numbers of digits permitted on R.H.S is 1 (digits in lower base)

Step 2: Cross subtraction: 98

<u>-1</u> 88

It is important to line the numbers as shown because 1 is not subtracted from 8 as usual but from 9 so as to get 88 as first part of answer.

Step 3: Vertical multiplication: $(-02) \times (-1) = 2$ (one digits allowed)

 \therefore Second part = 2

 $\therefore \qquad 98 \times 9 = 882$

Checking:

(Through 9 – check method)

98 = 8, 9 = 0, LHS = $98 \times 9 = 8 \times 0 = 0$

RHS = 882 = 8 + 8 + 2 = 18 = 1 + 8 = 9 = 0

As LHS = RHS So, correct answer

Example 2: 993 × 97

Step 1: 993 base is 1000 and deficiency is 007

97 base is 100 and deficiency is 03

 \therefore 993 – 007 (digits in lower base = 2 So, 2 digits are permitted

on

 \times 97 – 03 RHS or second part of answer)

Step 2: Cross subtraction:

993 <u>- 03</u> 963

Again line the number as shown because 03 is subtracted from 99 and not from 93 so as to get 963 which from first part of the answer.

Step 3: Vertical multiplication: (-007) - (-03) = 21 only two digits are allowed in the second part of answer So, second part = 21

 $\therefore \qquad 993 \times 97 = 96321$

Checking: (through 9 – check method)



9/93 = 3 97 = 7

- :. L.H.S. = $3 \times 7 = 21 = 2 + 1 = 3$ R.H.S. = 96321 = 2 + 1 = 3
- As LHS =RHS so, answer is correct

Example 3: 9996 base is 10000 and deficiency is 0004

988 base is 1000 and deficiency is 012

- \therefore 9996 0004 (digits in the lower base are 3 so,3 digits
 - \times 988 012 permitted on RHS or second part of answer)
- **Step 2 :** Cross subtraction:

9996 <u>- 012</u> 9876

Well, again take care to line the numbers while subtraction so as to get 9876 as the first part of the answer.

Step3 : Vertical multiplication: $(-0004) \times (-012) = 048$

(Remember, three digits are permitted in the second part i.e. second part of answer = 048

 \therefore 9996 × 988 = 9876048

Checking: (9 - check method) $\cancel{9996} = 6, \ 988 = 8 + 8 + = 16 = 1 + 6 = 7$ \Box LHS = $6 \times 7 = 42 = 4 + 2 = 6$ $|\text{RHS} = \cancel{9876045} = 8 + 7 = 15 = 1 + 5 = 6$ As, LHS =RHS so, answer is correct

When both the numbers are above base

Example 1: 105 × 12

Step 1: 105 base is 100 and surplus is 5

12 base is 10 and surplus is 2

 \therefore 105 + 05 (digits in the lower base is 1 so, 1 digit is permitted in the second part of answer) 12 + 2

Step 2: Cross – addition:

```
105 + 2 \\ 125
```

(again take care to line the numbers properly so as to get 125)

 \therefore First part of answer may be <u>125</u>



125 so as to get 126 $105 \times 12 = 1260$ **Checking:** 105 = 1 + 5 = 6, 12 = 1 + 2 = 3LHS = $6 \times 3 = 18 = 1 + 8 = 9 = 0$ RHS = 1260 = 1 + 2 + 6 = 9 = 0**Example 2:** 1122 × 104 **Step1:** 1122 – base is 1000 and surplus is 122 104 - base is 100 and surplus is 41122 + 122104 + 04 (digits in lower base are 2 so, 2-digits are permitted in the second part of answer) Step 2: Cross – addition 1122 (again take care to line the nos. properly so as to get 1162) 1162 + 04First part of answer may be 1162 **Step 3:** Vertical multiplication: $122 \times 04 = 4, 88$ But only 2 – digits are permitted in the second part, so, 4 is shifted to first part and added to 1162 to get 1166 (1162 + 4 = 1166) $1122 \times 104 = 116688$ Can be visualised as: 1122 + 122104 + 04 $1162 / \leftarrow (4) 88 = 116688$ +4/**Checking:** 1122 = 1 + 1 + 2 + 2 + = 6, 104 = 1 + 4 = 5 $LHS = 6 \times 5 = 30 = 3$ RHS = 116688 = 6 + 6 = 12 = 1 + 2 = 3LHS = RHS So, answer is correct As **Example 3:** 10007 × 1003 Now doing the question directly 10007 + 0007 base = 10000 $\times 1003 + 003$ base = 1000 10037 / 021 (three digits per method in this part) $10007 \times 10003 = 10037021$ 37 MATS Centre for Distance and Online Education, MATS University

Step 3: Vertical multiplication : $05 \times 2 = (1)0$ but only 1 digit is

permitted in the second part so 1 is shifted to first part and added to

...

...

...

...

....

...

...

....



```
Checking : 10007 = 1 + 7 = 8, 1003 = 1 + 3 = 4

\therefore LHS = 8 \times 4 = 32 = 3 + 2 = 5

RHS = 10037\ 021 = 1 + 3 + 1 = 5
```

As LHS = RHS so, answer is correct

Try These:

(i) 1015 × 103 (ii) 99888 × 91 (iii) 100034 × 102 (iv) 993 × 97 (v) 9988 × 98 (vi) 9995 × 96 (vii) 1005 × 103 (viii) 10025 × 1004 (ix) 102 × 10013 (x) 99994 × 95



Unit 6: Vinculum and Complimentary Techniques

VINCULUM: "Vinculum" is the minus sign put on top of a number e.g. $\overline{5}$, $4\overline{4}$, $6\overline{3}$ etc. which means (-5), (40 - 1), (60 - 3) respectively

Advantages of using vinculum:

- (1) It gives us flexibility, we use the vinculum when it suits us .
- (2) Large numbers like 6, 7, 8, 9 can be avoided.
- (3) Figures tend to cancel each other or can be made to cancel.
- (4) 0 and 1 occur twice as frequently as they otherwise would.

Converting from positive to negative form or from normal to vinculum form: Sutras: All from 9 the last from 10 and one more than the previous one

$$9 = 1$$
 1 (i.e. 10 – 1), $8 = 12$, $7 = 13$, $6 = 14$, $19 = 21$, $29 = 31$
 $28 = 32$, $36 = 44$ (40 – 4), $38 = 42$

Steps to convert from positive to vinculum form:

- (1) Find out the digits that are to be converted i.e. 5 and above.
- (2) Apply "all from 9 and last from 10" on those digits.
- (3) To end the conversions "add one to the previous digit".
- (4) Repeat this as many times in the same number as necessary.

Numbers with several conversions:

159 = 241 (i.e. 200 - 41)

168 = 232 (i.e. 200 - 32)

237 = 243 (i.e. 240 – 7)

1286 = 1314 (i.e. 1300 – 14)

2387129 = 24 1313 1 (here, only the large digits are be changed)

From vinculum back to normal form:

Sutras: "All from 9 and last from ten" and "one less than then one before".

```
1 \overline{1} = 09 (10 - 1), 1\overline{3} = 07 (10 - 3), 2\overline{4} = 16 (20 - 4), 2\overline{41} = 200 - 41 = 159, 1\overline{62} = 160 - 2 = 158

222 = 200 - 22 = 178 1314 = 1300 - 14 = 1286, 2413131 = 2387129 \text{ can be done in part as}

131 = 130 - 1 = 129 \text{ and } 241\overline{3} = 2400 - 13 = 2387

\therefore 24\overline{13}131 = 2387129.
```



Steps to convert from vinculum to positive form:

- (1) Find out the digits that are to be converted i.e. digits with a bar on top.
- (2) Apply "all from 9 and the last from 10" on those digits
- (3) To end the conversion apply "one less than the previous digit"
- (4) Repeat this as many times in the same number as necessary

Try These: Convert the following to their vinculum form: (i) 91 (ii) 4427 (iii) 183 (iv) 19326 (v) 2745 (vi) 7648 (vii) 81513 (viii) 763468 (ix) 73655167 (x) 83252327

Try These: From vinculum back to normal form. (i) 14 (i) 21 (iii) 23 (iv) 231 (v) 172 (vi) 1413 (vii) 2312132 (ix) 6322331 (x) 14142323

When one number is above and the other below the base

Example1: 102 × 97 Step 1: Here, base is 100 102 + 02(02 above base i.e. 2 surplus) 97 - 03(03 below base i.e. 3 deficiency) Step 2: Divide the answer in two parts as 102 / + 0297/-03Step 3: Right hand side of the answer is $(+02) \times (-03) = -06 = 06$ Step 4: Left hand side of the answer is 102 - 3 = 99 = 97 + 02 (same both ways) $102 \times 97 = 9906 = 9894$ (i.e. 9900 - 6 = 9894) *.*.. **Checking:** 102 = 1 + 2 = 3, 97 = 7*.*.. L.H.S. = $3 \times 7 = 21 = 1 + 2 = 3$ R.H.S = 9894 = 8 + 4 = 12 = 1 + 2 = 3· . As L.H.S. = R.H.S. So, answer is correct **Example 2 :** 1002 × 997 1002 + 002 (006 = 1000 - 6 = 994 and 1 carried from 999 to 999 reduces to 998) 997 - 003 999 006 $1002 \times 997 = 998\ 994$

When base is not same: Example1: 988×12



-012 base is 1000 deficiency 12 988 +2base is 10 surplus is 2, 1 digit allowed in R.H.S. 12 024 1188 - 2= 1186 = (2)4 $988 \times 12 = 11864 = 11856$ (because 4 = 10 - 4 = 6) *.*.. **Checking:** $9\ 88 = 8 + 8 = 16 = 1 + 6 = 7$, 12 = 1 + 2 = 3... LHS = $7 \times 3 = 21 = 2 + 1 = 3$ R.H.S = 11856 = 1 + 5 + 6 = 12 = 1 + 2 = 3LHS = RHS So, answer is correct As

Example 2: 1012 × 98

1012	1012	+ 012	(base is 1000, 12 surplus (+ve sign)
- 02	98	- 02	(base is 100, 2 deficiency (-ve sign)
992	992	-24	[As $012 \times (-02) = -24$] 2 digits allowed in RHS of

Answer

 \therefore 1012 × 98 = 99224 = 99176 [As 992200 - 24 = 99176]

Checking: 1012 = 1 + 1 + 2 = 4, 98 = 8LHS = $4 \times 8 = 32 = 3 + 2 = 5$ RHS = 99176 = 1 + 7 + 6 = 14 = 1 + 4 = 5As RHS = LHS so, answer is correct

Try These:

(i) 1015 × 89 (ii) 103 × 97 (iii) 1005 × 96 (iv) 1234 × 92 (v) 1223 × 92 (vi) 1051 × 9 (vii) 9899 × 87 (viii) 9998 × 103 (ix) 998 × 96 (x) 1005 × 107

Sub - base method:

Till now we have all the numbers which are either less than or more than base numbers. (i.e.10, 100, 1000, 10000 etc., now we will consider the numbers which are nearer to the multiple of 10, 100, 10000 etc. i.e. 50, 600, 7000 etc. these are called sub-base.

Example: 213 × 202

2

Step1: Here the sub base is 200 obtained by multiplying base 100 by



Step 2: R. H. S. and L.H.S. of answer is obtained using base- method.

213 + 13 <u>202</u> + 02 215 13 × 02 = 26 Step 3: Multiply L.H.S. of answer by 2 to get $215 \times 2 = 430$ \therefore 213 × 202 = 43026 \therefore

Example 2: 497 × 493

Step1: The Sub-base here is 500 obtained by multiplying base 100 by 5.

Step2: The right hand and left hand sides of the answer are obtained by using base method.

Step3: Multiplying the left hand side of the answer by 5.

		497	-03	
		493	-07	
Same		497–07 = 490	21	
		493 - 03 = 490		
	—	490×5		
		= 2450		
<i>.</i>		$497 \times 493 = 245021$		

Example 3: 206 × 197

Sub-base here is 200 so, multiply L.H.S. by 2

206	+ 06 - 03
197	- 03
206 - 3 = 203 $197 + 06 = 203 \times 2$ = 406	-18 = 18

 $\therefore 206 \times 197 = 4061\overline{8} = 40582$



Example 4: 212 × 188

Sub – base here is 200

$$212 + 12$$

$$188 -12$$

$$200 - 12 = 200 (1)44$$

$$188 + 12 = 200$$

$$- \times 2$$

$$400 - 1 = 399$$

 $\therefore 212 \times 188 = 39944 = 39856$

Checking:(11 – check method)

+-+

$$2 \ 1 \ 2 = 2 + 2 - 1 = 3$$

+-+
 $1 \ 8 \ 8 = 1 - 8 + 8 = 1$
L.H.S. = $3 \times 1 = 3$
+-+- +
R.H.S. = $3 \ 9 \ 8 \ 5 \ 6 = 3$
As L.H.S = R.H.S. So, answer is correct.

Try these

(1) 42 × 43 (2) 61 × 63 (3)8004 × 8012 (4)397 × 398 (5)583 × 593 (6) 7005 × 6998 (7) 499 × 502 (8) 3012 × 3001 (9)3122 × 2997 (10) 2999 × 2998

Doubling and Making halves

Sometimes while doing calculations we observe that we can calculate easily by multiplying the number by 2 than the larger number (which is again a multiple of 2). This procedure in called doubling:

$$35 \times 4 = 35 \times 2 + 2 \times 35 = 70 + 70 = 140$$

$$26 \times 8 = 26 \times 2 + 26 \times 2 + 26 \times 2 + 26 \times 2 = 52 + 52 + 52 + 52$$

$$= 52 \times 2 + 52 \times 2 = 104 \times 2 = 208$$



$53 \times 4 = 53 \times 2 + 53 \times 2 = 106 \times 2 = 212$

Sometimes situation is reverse and we observe that it is easier to find half of the number than calculating 5 times or multiples of 5. This process is called

Making halves:

4. (1) 87 × 5 = 87 × 5 × 2/2 = 870/2 = 435
(2) 27 × 50 = 27 × 50 × 2/2 = 2700/2 = 1350
(3) 82 × 25 = 82 × 25 × 4/4 = 8200/4 = 2050

Try These:

 18×4 (1) 14×18 (2) (3) 16×7 (4) 16×12 (5) 52×8 68×5 (6) 36×5 (7) 46×50 (8) (9) 85×25 $(10) 223 \times 50$ (11) 1235×20 (12) 256 × 125 (13) 85 × 4 $(14)\ 102 \times 8$ $(15) 521 \times 25$

Multiplication of Complimentary numbers :

Sutra: By one more than the previous one.

This special type of multiplication is for multiplying numbers whose first digits(figure) are same and whose last digits(figures)add up to 10,100 etc.

Example 1: 45×45 **Step I:** $5 \times 5 = 25$ which form R.H.S. part of answer **Step II:** $4 \times$ (next consecutive number)

> i.e. $4 \times 5 = 20$, which form L.H.S. part of answer $\therefore 45 \times 45 = 2025$



Example 2: $95 \times 95 = 9 \times 10 = 90/25 \longrightarrow (5^2)$ i.e. $95 \times 95 = 9025$ Example 3: $42 \times 48 = 4 \times 5 = 20/16 \longrightarrow (8 \times 2)$ $\therefore \quad 42 \times 48 = 2016$ Example 4: $304 \times 306 = 30 \times 31 = 930/24 \longrightarrow (4 \times 6)$ $\therefore \quad 304 \times 306 = 93024$

Try These:

(1) 63×67 (2) 52×58 (3) 237×233 (4) 65×65 (5) 124×126 (6) 51×59 (7) 762×768 (8) 633×637 (9) 334×336 (10) 95×95

Multiplication by numbers consisting of all 9's :

Sutras: 'By one less than the previous one' and 'All from 9 and the last from 10'

When number of 9's in the multiplier is same as the number of digits in the multiplicand.

Example 1 : 765 × 999

Step I : The number being multiplied by 9's is first reduced by 1

i.e. 765 - 1 = 764 This is first part of the answer

Step II : "All from 9 and the last from 10" is applied to 765 to get

235, which is the second part of the answer.

 $\therefore 765 \times 999 = 764235$

When 9's in the multiplier are more than multiplicand

Example II : 1863 × 99999

Step I : Here 1863 has 4 digits and 99999 have 5-digits, we suppose 1863 to be as 01863. Reduce this by one to get 1862 which form the first part of answer.



Step II: Apply 'All from 9 and last from 10' to 01863 gives 98137which form the last part of answer \therefore 1863 x 99999 = 186298137

When 9's in the multiplier are less than multiplicand

Example 3 : 537 x 99

Step I: Mark off two figures on the right of 537 as 5/37, one more than the L.H.S. of it i.e. (5+1) is to be subtracted from the whole number, 537 - 6 = 531 this forms first part of the answer Step II: Now applying "all from 9 last from 10" to R.H.S. part of 5/37 to get 63 (100 - 37 = 63) \therefore 537 x 99 = 53163

Try these

(1)254 × 999 (2)7654 × 9999 (3)879 × 99 (4)898 × 9999 (5)423 × 9999 (6)876 × 99 (7)1768 × 999 (8)4263 × 9999 (9)30421 × 999 (10)123 × 99999



Unit 7: Right-to-Left Multiplication Strategies

Multiplication by 11

Example 1: 23 × 11

Step 1 : Write the digit on L.H.S. of the number first. Here the

number is 23 so, 2 is written first.

Step 2 : Add the two digits of the given number and write it in

between. Here 2 + 3 = 5

Step 3 : Now write the second digit on extreme right. Here the digit is 3. So, $23 \times 11 = 253$

OR

23 × 11 = 2 / 2+3 / 3 = 253

(Here base is 10 so only 2 digits can be added at a time)

Example 2: 243 × 11

Step 1: Mark the first, second and last digit of given number First

digit = 2, second digit = 4, last digit = 3

Now first and last digits of the number 243 form the first and last digits of the answer.

Step 2: For second digit (from left) add first two digits of the number

i.e. 2 + 4 = 6

Step 3: For third digit add second and last digits of the number

i.e. 3 + 4 = 7

So, $243 \times 11 = 2673$

OR

243 × 11 = 2 / 2 + 4 / 4 + 3 / 3 = 2673

Similarly we can multiply any bigger number by 11 easily.

Example 3: 42431 × 11

42431 × 11 = 4 / 4 + 2 / 2 + 4 / 4 + 3 / 3 + 1 / 1 = 466741

If we have to multiply the given number by 111

Example 1: 189 × 111

Step 1: Mark the first, second and last digit of given number First

digit = 1, second digit = 8, last digit = 9

Now first and last digits of the number 189 may form the first and last digits of the answer

Step 2: For second digit (from left) add first two digits of the number i.e. 1 + 8 = 9

Step 3: For third digit add first, second and last digits of the number to get 1 + 8 + 9 = 18 (multiplying by 111, so three digits are

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added at a time)

Step 4: For fourth digit from left add second and last digit to get, 8 + 9 = 17

As we cannot have two digits at one place so 1 is shifted and added to the next digit so as to get $189 \times 111 = 20979$

OR

∴189 × 111 = 20979

Example 2 : 2891 × 111

$$2 | 2+8 | 2+8+9 | 8+9+1 | 9+1$$

$$10+2 = 19+1 | 18+1 = 1 0 1$$

$$=1 2 = 20 = 19$$

$$2891 \times 111 = 320901$$

Note: This method can be extended to number of any size and to multiplying by 1111, 11111 etc. This multiplication is useful in percentage also. If we want to increase a member by 10% we multiply it by 1.1



General Method of Multiplication.

Sutra: Vertically and cross-wise.

Till now we have learned various methods of multiplication but these are all special cases, wherenumbers should satisfy certain conditions like near base, or sub base, complimentary to each other etc. Now we are going to learn about a general method of multiplication, by which we can multiply any two numbers in a line. Vertically and cross-wise sutra can be used for multiplying any number.

For different figure numbers the sutra works as follows:

Two digit – multiplication

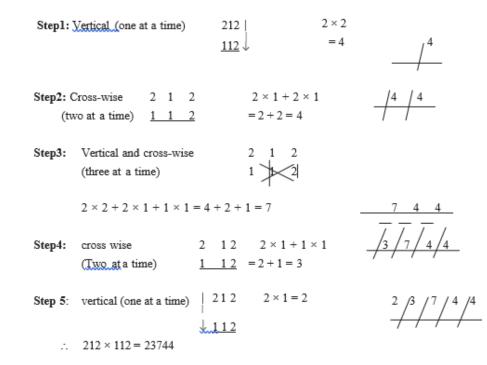
Example: Multiply 21 and 23

FFF		
Step1: Vertical (one at a time)	$2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} 1 \times 3 = 3$	
_	↓	
Step2: Cross -wise (two at a time)	2 1	$(2 \times 3 + 2 \times 1) = 8$
	2 3	83
Step3: Vertical (one at a time)	21 1	2 × 2 = 4
	2] 3	483
_		
∴ 21 × 23 = 483		
Multiplication with carry:		
Example: Multiply 42 and 26		
	$2 \times 6 = 12$	1 12
Step1: Vertical 42 $26 \downarrow$	20 12	
<u>20</u> \		
Step2: Cross-wise 4 2	$4 \times 6 + 2 \times 2$	0 1
2 6	24 + 4 = 28	
	_	
Step3: Vertical 42 4×2	2 = 8	8 8 2
26		+2 1) = 10 = 2(9)
		- 10 - 20
		- 10 - 49
∴ 42 × 26 = 1092		

Three digit multiplication: Example: 212 × 112

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Three digits Multiplication with carry:

Example: 816 × 223

$$\therefore 816 \times 223 = 181968|$$
Checking by 11 - check method
+-+ -+
 $816 = 14 - 1 = 13 = 3 - 1 = 2$
+-+
 $223 = 3$
 \therefore L.H.S. = $3 \times 2 = 6$
 $- + - + - +$
 181968 = $- +$
 481968 = $- +$
 $17 = 7 - 1 = 6$
As L.H.S. = R.H.S.
 \therefore Answer is correct

Try These:

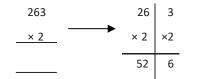
- (1) 342×514 (2)1412 × 4235 (3)321 × 53 (4) 2121 × 2112
- (5) 302×415 (6) 1312×3112 (7) 5123×5012 (8) 20354×131
- (9) 7232 × 125 (10) 3434 × 4321



Number Split Method

As you have earlier used this method for addition and subtraction, the same may be done for multiplication also.

For example :

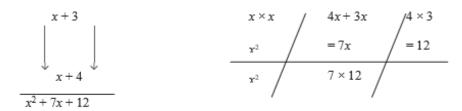


Note : The split allows us to add 36 + 24 and 42 + 39 both of which can be done mentally

Multiplication of algebraic expressions:

Sutra: Vertically and cross-wise

Example1 : (x + 3) (x + 4)



Example2: (2x + 5)(3x + 2)

2x	+	5		
		!		
ļ				_
		Ļ		
3x	+	2		

$(2x) \times (3x)$	4x + 15x	10
=6x ²	= 19x	

 $6x^2 + 19x + 10$

Example3: $(x^2 + 2x + 5) (x^2 - 3x + 1)$

Try These:

(1) (2x - 1) (3x + 2) (2) (2x + 1) (x2 + 3x - 5) (3) (5x + 5) (7x - 6) (4) (x + 5) (x2 - 2x + 3) (5) (x - 4) (x2 + 2x + 3) (6) (x2 + 4x - 5) (x + 5) (7) (x3 - 5) (x2 + 3)

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O Notes

 $\begin{array}{ll} (8) & (x2-2x+8) (x4-2) \\ (9) & (x2-7x+4) (x3-1) \\ (10) (x3-5x2+2) (x2+1) \end{array}$



MODULE 4 Vedic Methods for Squaring and Division

LEARNING OUTCOMES

By the end of this module, learners will be able to:

- Square numbers quickly using techniques like base method and pattern recognition.
- Estimate and calculate square roots mentally for perfect and near-perfect squares.
- Perform division using special Vedic methods like straight division and pattern-based shortcuts.
- Understand the logic behind Vedic squaring and division for deep conceptual clarity.
- Enhance overall mental math ability with advanced squaring and division strategies.



Unit 8: Squaring

Square of numbers ending in 5 :

Sutra: 'By one more than previous one"

Example: 75×75 or 75^2

As explained earlier in the chapter of multiplication we simply multiply 7 by the next number i.e. 8 to get 56 which forms first part of answer

and the last part is simply $25=(5)^2$. So, $75 \times 75 = 5625$

This method is applicable to numbers of any size.

Example: 605²

 $60 \times 61 = 3660$ and $5^2 = 25$

 $\therefore 605^2 = 366025$

Square of numbers with decimals ending in 5

Example : $(7.5)^2$

 $7 \times 8 = 56, (0.5^2) = 0.25$

 $(7.5)^2 = 56.25$ (Similar to above example but with decimal) Squaring numbers above 50:

Example: 52²

Step1: First part is calculated as $5^2 + 2 = 25 + 2 = 27$ Step2: Last part is calculated as $(2)^2 = 04$ (two digits) $\therefore 52^2 = 2704$

Squaring numbers below 50

Example : 48^2 Step1: First part of answer calculated as: $5^2 - 2 = 25 - 2 = 23$ Step2: second part is calculated as : $2^2 = 04$ \therefore $48^2 = 2304$

Squaring numbers near base :

Example : 1004^2 **Step1:** For first part add 1004and 04 to get 1008 **Step2:** For second part4² = 16 = 016 (as,base is 1000 a three digit no.) \therefore (1004)² = 1008016



Squaring numbers near sub - base:

Example (302)² Step1: For first part = 3 (302 + 02) = 3 × 304 = 912 [Here sub – base is 300 so multiply by 3] Step2: For second part = 2² = 04

 \therefore (302)² = 91204

General method of squaring:

The Duplex

Sutra: "Single digit square, pair multiply and double" we will use the term duplex, D' as follows:

For <u>1 figure</u>(or digit) Duplex is its squaree.g. $D(4) = 4^2 = 16$ For<u>2 digits</u>Duplex is twice of the product e.g. $D(34) = 2 (3 \times 4) = 24$ For <u>3 digit number</u>: e.g. $(341)^2$ $D(3) = 3^2 = 9$ $D (34) = 2 (3 \times 4) = 24$ $D (341) = 2 (3 \times 1) + 4^2 = 6 + 16 = 22$ p / 4 / 2 / 8 / 1 p / 4 / 2 / 1 p / 4 / 2 / 1 p / 4 / 2 / 1 p / 4 / 2 / 1 p / 4 / 2 / 1 p / 4 / 2 / 1 p / 1

Algebraic Squaring :

Above method is applicable for squaring algebraic expressions:

Example: $(x + 5)^2$ D $(x) = x^2$ D $(x + 5) = 2 (x \times 5) = 10x$ D $(5) = 5^2 = 25$ $\therefore (x + 5)^2 = x^2 + 10x + 25$ Example: $(x - 3y)^2$ D $(x) = x^2$ D $(x - 3y) = 2 (x) \times - 3y) = -6xy$ D $(-3y) = (-3y)^2 = 9y^2$ $\therefore (x - 3y)^2 = x^2 - 6xy + 9y^2$



Try these:



Unit 9: Square Roots

General method:

As $1^2 = 1\ 2^2 = 4\ 3^2 = 9\ 4^2 = 1[\ 6\]\ 5^2 = 2\ [5]\ 6^2 = 3\ [6]$ $7^2 = 4\ [9]\ 8^2 = 6\ [4]\ 9^2 = 8[1]$ i.e. square numbers only have digits 1,4,5,6,9,0 at the units place (or at the end) Also in 16, digit sum = 1 + 6 = 7, 25 = 2 + 5 = 7, 36 = 3 + 6 = 9, 49 = 4 + 9 = 13 $13 = 1 + 3 = 4,\ 64 = 6 + 4 = 10 = 1 + 0 = 1,\ 81 = 8 + 1 = 9$ i.e. square number only have digit sums

This means that square numbers cannot have certain digit sums and they cannot end with certain figures (or digits) using above information which of the following are not square numbers:

(1) 4539 (2) 6889 (3) 104976 (4) 27478 (5) 12345

Note: If a number has a valid digit sum and a valid last figure that does not mean that it is a square number. If 75379 is not a perfect square in spite of the fact that its digit sum is 4 and last figure is 9.

Square Root of Perfect Squares:

Example1: $\sqrt{5184}$

Step 1: Pair the numbers from right to left <u>5184</u> two pairs Therefore answer is 2 digit numbers

 $7^2 = 49$ and $8^2 = 64$

49 is less than 51

Therefore first digit of square root is 7. Look at last digit which is 4

As $2^2 = 4$ and $8^2 = 64$ both end with 4 Therefore the answer could be

72 or 78 As we know $75^2 = 5625$ greater than 5184

Therefore $\sqrt{5184}$ is below 75

Therefore $\sqrt{5184}=72$

Example 2: √9216

Step 1: Pair the numbers from right to left <u>9216</u> two pairs Therefore answer is 2 digit numbers

 $9^2 = 81$ and $10^2 = 100$

81 is less than 92

Therefore first digit of square root is 9. Look at last digit which is 6

As $4^2 = 16$ and $6^2 = 36$ both end with 6

Therefore the answer could be 94 or 96



As we know $95^2 = 9025$ less than 9216Therefore $\sqrt{9216}$ is above 95Therefore $\sqrt{9216} = 96$

General method

Example 1 : $\sqrt{2809}$

Step1: Form the pairs from right to left which decide the number of digits in the square root. Here 2 pairs therefore 2 - digits in thesquare root

Step 2: Now $\sqrt{28}$, nearest squares is = 25 So first digit is 5 (from left)

Step3: As 28 - 25 = 3 is reminder which forms 30 with the next digit 0.

Step 4: Multiply 2 with 5 to get 10 which is divisor $10 \sqrt{2809}$ 30

> Now $3 \times 10 = 30 \ \underline{30} = Q \ R$ 10 3 0

Step 5: As $3^2 = 9$ and 9 - 9 (last digit of the number) = 0 \therefore 2809 is a perfect square and $\sqrt{2809} = 53$

Example 2: $\sqrt{3249}$

Step1: Form the pairs form right to left which decided the number of digits in the square root. Here 2 pairs therefore 2 digits in the square root.

Step2: Now $32 > 25 = 5^2$ so the first digit in 5 (from left)

Step 3: 32 - 25 = 7 is remainder which form 74 with the next digit 4

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Step 4: Multiply 2 with 5 to get 10 which is divisor $10\sqrt{3249}$ Now <u>74</u> = Q R 74

107 4

Step5: $7^2 = 49$ and 49 - 49 = 0 (remainder is 4 which together with9 form49)

Example 3: $\sqrt{54\ 75\ 6}$

Step1: Form the pairs from right to left therefore the square root of 54756 has 3-digits.

Step2: $5 > 4 = 2^2$ i.e. nearest square is $2^2 = 4$ So first digit is 2 (from left)

 $[\]therefore$ 3249 is a perfect square and $\sqrt{3249} = 57$



Step3: As 5 - 4 = 1 is remainder which form 14 with the next digit 4. Step4: Multiply 2 with 2 to get 4, which is divisor 2 4 $5_{1}4_{-2}75.6$ Now 14 = Q R4 3 2 Step 5: Start with remainder and next digit, we get 27. Find $27 - 3^{2} = 27 - 9 = 18$ [square of quotient] 234 Step 6: $18 = Q R 4 5_{1}4_{2}7_{2}5_{1}6$ 4 4 2 Now $25 - (3 \times 4 \times 2) = 25 - 24 = 1$ 1 = Q R4 0 1 $16 - 4^{2} = 16 - 16 = 0$

 \therefore 54756 is a perfect square and so $\sqrt{5}$ 4 7 5 6 = 234

Try These:

1.	2116	2.	784
3.	6724	4.	4489
5.	9604	6.	3249
7.	34856	8.	1444
9.	103041	10.	97344

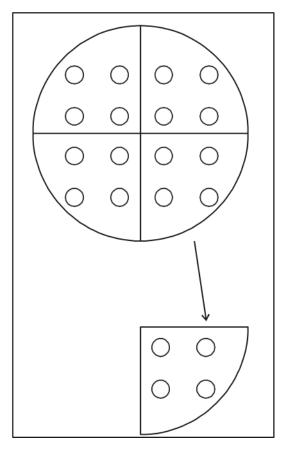


Unit 10: Division

Defining the Division terms

There are 16 balls to be distributed among 4 people How much each one will get is a problems of division.

Let us use this example to understand the terms used in division.



Divisor: —Represent number of people we want to distribute them or the number that we want to divide by. Here the divisor is 4. **Dividend:** -Represents number of balls to be divided 16 in this case. **Quotient:** Represents the number of balls in each part, 4 is this case. **Remainder:** What remains after dividing in equal parts, 0 in this

case?

The remainder theorem follows from the division example above and is expressed mathematically as follows.

 $Divided = Divisor \times Quotient + Remainder$

The remainder theorem can be used to check the Division sums in Vedic Mathematics as described in the following sections.



Different methods are used for dividing numbers based on whether the divisor is single digit numbers below a base, above a base or no special case.

Special methods of Division.

Number splitting

Simple Division of Divisor with single digits can be done using this method. **Example:** The number 682 can be split into 6/82 and we get 3/41 because 6 and 82 are both easy to halve Therefore 682/2 = 341 **Example:** 3648/2 becomes 36/48/2 = 18/24 = 1824 **Example:** 1599/3 we notice that 15 and 99 can be separately by 3 so 15/99/3 = 5/33 = 533 **Example:** 618/6 can also be mentally done 6/18/6 = 103 note the 0 here Because the 18 takes up two places **Example:** 1435/7 14/35/7 = 2/05 = 205 **Example:** 27483/3 becomes 27/48/3/3 = 9/16/1 = 9161

Practice Problem

Divided mentally (Numbers Splitting)

- (1) 2)656
- (2) 2)726
- (3) 3)1899
- (4) 6)1266
- (5) 3)2139
- (6) 2)2636
- (7) 4)812
- (8) 6)4818
- (9) 8)40168
- (10) 5)103545

Division by 9

As we have seen before that the number 9 is special and there is very easy way to divide by 9.

Example : Find 25 ÷ 9



25/9 gives 2 remainder 7

The first figure of 25 is the answer?

And adding the figures of 25 gives the remainders 2 + 5 = 7 so $25 \div 9 = 2$ remainder 7. It is easy to see why this works because every 10 contains 9 with 1 left over, so 2 tens contains 2 times with 2 left over. The answer is the same as the remainders 2. And that is why we add 2 to 5 to get remainder. It can happen that there is another nine in the remainder like in the next example

Example: Find 66 ÷ 9

66/9 gives 6 + 6 = 12 or 7 or 3

We get 6 as quotient and remainder 12 and there is another nine in the remainder of 12, so we add the one extra nine to the 6 which becomes 7 and remainder is reduced to 3 (take 9 from 12)

We can also get the final remainder 3, by adding the digits in 12. The unique property of number nine that it is one unit below ten leads to many of the very easy Vedic Methods.

This method can easily be extended to longer numbers.

Example: $3401 \div 9 = 377$ remainder 8

Step 1: The 3 at the beginning of 3401 is brought straight into the answer.

9)3401

Step 2: This 3 is add to 4 in 3401 and 7 is put down

9)3401

<u>37</u>

3

Step 3: This 7 is then added to the 0 in 3401 and 7 is put down.

9)3401

<u>377</u>

Step 4: This 7 is then added to give the remainder 9) 340/1

377/8

Divided the following by 9



(1) 9)51 9)34 (2)(3) 9)17 (4) 9)44 (5) 9)60 (6) 9)26 9)46 (7)9)64 (8) 9)88 (9)

(10) 9)96

Longer numbers in the divisor

The method can be easily extended to longer numbers. Suppose we want to divide the number 21 3423 by 99. This is very similar to division by 9 but because 99 has two 9's we can get the answer in two digits at a time. Think of the number split into pairs.

21/34/23 where the last pair is part of the remainder.

Step 1: Then put down 21 as the first part of the answer

```
99)21/34/23
```

21

Step 2: Then add 21 to the 34 and put down 55 as next part

```
99)21/34/23
```

Step 3: Finally add the 55 to the last pair and put down 78 as the remainder

So the answer is 2155 remainder 78

Example: 12/314 ÷ 98 = 1237

Step 1: This is the same as before but because 98 is 2 below 100 we double the last part of the answer before adding it to the next part of the sum. So we begin as before by bringing 12 down into the answer.

Step 2: Then we double 12 add 24 to 13 to get 37



98) 12/13/14

```
12/37
```

Step 3: Finally double 37 added $37 \times 2 = 74$ to 14

98)12/13/14

<u>12/37/88</u> =1237 remainder 88.

It is similarly easy to divide by numbers near other base numbers 100, 1000 etc.

Example: Suppose we want to divide 236 by 88 (which is close to 100). We need to know how many times 88 can be taken from 235 and what the remainder is

Step 1: We separate the two figures on the right because 88 is close to 100 (Which has 2 zeros)

88) 2/36

Step 2: Then since 88 is 12 below 100 we put 12 below 88, as shown

88) 2/36

Step 3: We bring down the initial 2 into the answer

88) 2/36

12

<u>2</u>

Step 4: This 2 is multiplies Haggled 12 and the 22 is placed under the 36 as Shown

88) 2/36
12 2 / 24
Step 5: We then simply add up the last two columns.
88) 2/36
12 2 r 60

In a similar way we can divide by numbers like 97 and 999.

Practice problems

Divide the following using base method

- (1) 121416 by 99
- (2) 213141 by 99
- (3) 332211 by 99
- (4) 282828 by 99
- (5) 363432 by 99



(6) 11221122 by 98(7) 3456 by 98

Sutra: Transpose and Apply

A very similar method, allows us to divide numbers, which are close to but above a base number.

Example: 1479 ÷ 123 = 12 remainder 13
Step 1: 123 is 23 more than base 100
Step 2: Divide 1479 in two columns therefore of 2digit each
Step 3: Write 14 down
Step 3: Write 14 down

- **Step 4:** Multiply 1 by 23 and write it below next two digits. Add in the Second column and put down 2.
- Step 5: Add multiply this 2 the 2, 3 and put 46 then add up last two Columns

Straight Division

The general division method, also called Straight division, allows us to divide numbers of any size by numbers of any sine, in one line, Sri BharatiKrsnaTirthaji called this "the cowing gem of Vedic Mathematics"

Sutra: - 'vertically and crosswise' and 'on the flag'

Example: Divide 234 by 54

The division, 54 is written with 4 raised up, on the flag, and a vertical line is drawn one figure from the right hand end to separate the answer, 4, from the remainder 28

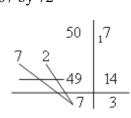
$$\begin{array}{c}
23 \\
5^{4}20 \\
4 \\
28
\end{array}$$

Step 1: 5 into 20 goes 4 remained 3 as shown

Step 2: Answer 4 multiplied by the flagged 4 gives 16 and this 16 taken from 34 leaves the remainder 28 as shown



Example: Divide: 507 by 72



Step 1: 7 into 50 goes 7 remainder 1 as shownStep 2: 7 times the flagged 2 gives 14 which we take from 17 to have remainder of 3

Split Method

Split method can be done for division also. For example :

6234 ÷ 2	62	34
	62 ÷ 2	÷2
	31	17

The 'split' may require more 'parts'.

30155 ÷ 5	30	15		
	÷5	÷5	÷	
			5	
	6	0 3	1	6031
		3		
244506 ÷ 3	24	45	06	
	÷3	÷3	÷3	
	8	1 5	02	81502
		5		

Practice Question

Divide the following using straight division

(1) $209 \div 552$	(2)	621 ÷ 63
(3) $503 \div 72$	(4)	103 ÷ 43
(5) $74 \div 23$	(6)	504 ÷ 72
(7) $444 \div 63$	(8)	543 ÷ 82
(9) 567÷93	(10)	97 ÷ 28
(11) 184 ÷ 47	(12)	210 ÷ 53
(13) 373 ÷ 63	(14)	353 ÷ 52
(15) 333 ÷ 44	(16)	267 ÷ 37



Books for Reference

1.Sri BharatiKrsnaTirthaji, "Vedic Mathematics", published by MotilalBanarsidass, 1965. ISBN 81- 208-0163-6.

2.Williams K.R. "Discover Vedic Mathematics." Vedic Mathematics Research Group, 1984. ISBN 1- 869932-01-3

3.Williams K.R. and M. Gaskell "The Cosmic Calculator". MotilalBanarsidass, 2002.ISBN 81-208- 1871-7.

4.Nicholas A.P., Williams, J. Pickles. "Vertically and Crosswise". Inspiration Books, 1984. ISBN 1- 902517-03-2.

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